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Some Aspects of Measurement Error in the United States Objective Yield Survey[1]

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Abstract: In this paper we consider aspects of measurement error in the United States Objective Yield Survey for corn, which is conducted by the United States Department of Agriculture National Agriculture Statistics Service. Various models are used to assess measurement error in variables that appear in forecasting models for end-of-season yield. Two major components of yield are number of ears (per sample unit) and average grain weight per ear. Variables that are used in forecasting number of ears are simple counts, such as number of stalks, and have very high reliability ratios. The reliabilities for variables that are used in forecasting grain weight, although adequate, are substantially lower than those found for number of ears. We also consider a model for the reliability of yearly means, and discuss the effect of measurement error on end-of-season forecasts.

Key words: Agricultural statistics; forecasting; reliability; structural equations; LISREL.

1. Introduction

The Objective Yield Survey is a series of monthly measurements conducted by the United States Department of Agriculture (USDA) National Agricultural Statistics Service (NASS) during the growing season for the purpose of forecasting end-of-season yield for crops such as corn, soybeans and wheat. In this paper, we discuss models for corn yield using data from the state of Iowa during the years 1979 to 1985. Thorough discussions of the survey design and current forecast methods can be found in Francisco, Fuller, and Fecso (1987), and Reiser, Fecso, and Taylor (1987), respectively. An abbreviated description follows.

An early season estimate for the number of acres planted or to be planted to corn is calculated by NASS from data collected during the June Enumerative Survey, using a multistage stratified area sample. Objective Yield Surveys are conducted during the months of July through November, using fields that are subsampled from those visited during the June Enumerative Survey. Even though the data represent observations over time, the number of time points per year is small, and, in the past, the data have been treated as essentially cross sectional. That is, data from July are used to develop a July forecasting model, data from August are used to develop an August forecasting

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model, etc., and trends over months within years are not presently used. In this paper we consider the use of panel models, which incorporate aspects of both cross sectional and time series data. Panel models are heavily used in the social sciences, where the emphasis is on the process of change over time – i.e., which variables influence other variables over time. Here, the use of panel models has a different emphasis: The models will be used for the purpose of identifying measurement error in the predictor variables, and, ultimately, to assess the effect of that measurement error on forecasts. This information would be useful for determining at which stage of growth a measurement provides useful information. Thus, efficiencies in data collection could be achieved by elimination of field procedures which do not substantially improve forecasting ability.

In the following section we present a brief overview of the forecasting methods presently used by the USDA. Then, in Section 3 we consider a single indicator panel model for number of ears of corn, and in Section 4 we consider a two-variable, two-wave model for size of ears.

2. Present Methods

In Iowa, approximately 240 fields are selected each year for the Objective Yield Survey. Within each selected field, a pair of randomly located units is established for data collection. Each unit is two rows (of corn) wide and fifteen feet long. In early July (month number one for the Objective Yield Survey), one-half of the selected fields are visited, and data pertaining to number of stalks in each unit are collected. Starting in August (month two), all selected fields are visited monthly until the crop is either harvested or fully mature. No fields are visited after November (month five). During the visits in months two through five, data are collected on both number of ears and size of ears. Although 240 units are selected for the sample each year, some units are lost to refusals by owners, changes from intentions to plant as stated in June, and damage to crops. As a result, data are available each year on approximately 200 fields, or 200 pairs of units. For analysis purposes, each pair of units is treated as a single observation.

End of season yield is calculated for a selected field using total number of ears and average grain weight per ear

$$Y_{ij} = Y_{wij} Y_{Nij} K S_{ij}^{-1} \quad (1)$$

Y_{ij} = yield for field j in year i in bushels per acre

Y_{wij} = Average grain weight per ear, for field j in year i, measured on the ears in the selected units

Y_{Nij} = Number of ears of corn in the selected units for field j in year i

K = a constant for the transformation to bushels per acre = 103.714

S_{ij} = width of eight rows of corn, field ij.

During the growing season, the yield for a selected field is calculated from expression (1) using forecasted values for grain weight or number of ears, or both. Forecast models use early season plant characteristics to predict end-of-season yield in terms of number of ears and grain weight per ear. The model for predicting number of ears at the end of season uses number of stalks and number of ears measured earlier in the season as predictors

$$Y_N = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon \quad (2)$$

where

Y_N = end-of-season number of ears

X_1 = number of ears with kernels (months 2 and 3 only)

X_2 = number of stalks

X_3 = number of stalks with ears

X_4 = number of ears or ear shoots.

The model for grain weight uses two measures of ear length

$$Y_W = \beta_0 + \beta_1 X_5 + \beta_2 X_6 + \varepsilon \quad (3)$$

where

Y_W = average grain weight per ear, at end of season

X_5 = average length over husk

X_6 = total length of five kernel rows.

2.2. *Maturity class*

At any given point in the growing season, there is considerable field to field variation in the maturity of corn due to differences in timing of spring planting, rainfall, etc. Previous experience has indicated that corn which is behind in maturity early in the season does not catch up with the more mature corn by the end of the season. As a result, the more mature corn at any given time during the season will tend to have a higher end-of-season yield. Therefore, the concept of *maturity class* has been introduced into forecasting end-of-season yield.

Each field from which measurements are taken is assigned to one of six maturity classes based on characteristics of the corn plants. For example, if the ears of corn have no silks showing, then the corn is placed in maturity class one. If silks are showing, but little or no watery liquid is present in the spiklets, the corn is at maturity stage two. At maturity stage six, ears are firm and solid, kernels are fully dented with no milk present in most kernels; the shucks are dry, but not beginning to open up. Finally, at maturity stage seven, the corn is fully mature.

A different forecasting model has been used for each maturity class that is available within a month, because more mature plants should have a model (i.e., different parameter values) that reflects higher end-of-season yield. The various models follow expressions (1), (2), and (3), but the notation would require two additional subscripts, one for month and one for maturity class. Some predictor variables are not available in early maturity classes, and not all matur-

Table 1. Availability of variables by month (Number of observations for years 1979–85 combined).

Variable	Month 1	2	3	4
Number of stalks	702	1212	1405	NA
Number of stalks with ears	714	1212	1405	NA
Number of ears or ear stalks	714	1212	1405	NA
Number of ears with kernels	NA	1196	1405	1405
Average length over husk	NA	1191	687	NA
Total length five kernel rows	NA	1192	697	NA

Table 2. *OLS parameter estimates for number of ears*

Maturity stage	n	$\hat\beta_0$	$\hat\beta_1$	$\hat\beta_2$	$\hat\beta_3$	$\hat\beta_4$	$\hat\sigma^2_e$	R^2
				Month 1				
1	118	10.57 (3.684)	–	0.836** (0.045)	–	–	63.62	0.075
2	545	13.199** (2.451)	–	0.783** (0.0306)	−0.212** (0.051)	0.193* (0.034)	114.64	0.60
Maturity stage	n	$\hat\beta_0$	$\hat\beta_1$	Month 2 $\hat\beta_2$	$\hat\beta_3$	$\hat\beta_4$	$\hat\sigma^2_e$	R^2
2 & 3	58	8.819 (3.427)	−0.031 (0.054)	0.067 (0.125)	0.642** (0.156)	0.179** (0.051)	25.45	0.89
4	574	1.372 (0.935)	0.687** (0.029)	−0.074* (0.038)	0.315** (0.050)	0.038** (0.011)	16.29	0.93
5	446	−0.322 (1.001)	0.872** (0.030)	0.021 (0.038)	0.055 (0.049)	0.037** (0.011)	14.10	0.95
6	121	0.703 (1.878)	0.831* (0.044)	0.007 (0.034)	0.074 (0.055)	0.060* (0.017)	9.74	0.96
Maturity stage	n	$\hat\beta_0$	$\hat\beta_1$	Month 3 $\hat\beta_2$	$\hat\beta_3$	$\hat\beta_4$	$\hat\sigma^2_e$	R^2
5	69	0.946 (1.482)	0.952** (0.064)	0.017 (0.0697)	0.008 (0.103)	0.002 (0.02)	5.455	0.981
6	616	0.884 (0.524)	0.938** (0.017)	0.0003 (0.019)	0.043* (0.026)	0.002* (0.0056)	5.120	0.977

*$p < 0.05$
**$p < 0.01$
X_1 = number of ears with kernels (month 2 and 3)
X_2 = number of stalks
X_3 = number of stalks with ears
X_4 = number of ears or ear shoots

ity classes are present in every month. For example, in month one (July) corn never matures beyond maturity stage two, and so there are only two models for number of ears in month one. Also, none of the measurements for size of corn ears, which are the variables for predicting grain weight, are available during either maturity stage one or two, so in month one grain weight is forecast by using the historical average from the previous five years. The availability of predictor variables by month is shown in Table 1.

2.3 Estimation

Parameter estimates for the forecasting models are established by using measurements from several years preceding the current forecast year. The dependent variable is end-of-season yield, and the predictor variables are early season measurements. Estimation is performed by ordinary least squares. Parameter estimates based on data from 1979 to 1985 are shown by month and maturity class in Tables 2 and 3. The USDA actually calculates estimates for year i using data from the preceding five years (i-1 to i-5). There are two competing principles behind this choice for number of years on which to base the estimates: to reduce sampling error, we would like to use data from as many years as possible; however, technological innovations may produce trends that render data from past years obsolete. So the USDA uses only the previous five years as a compromise between these two principles. The estimates shown in Tables 2 and 3, however, are based on seven years of data.

Two aspects of the data that are not incorporated into the OLS estimation include stratification of the samples on which the data are collected, and secondly, year-to-year effects on yield. Incorporating stratification into the estimators would produce only small differences, because the design of

Table 3. *OLS parameter estimates for grain weight*

Maturity stage	n	$\hat\beta_0$	Month 2 $\hat\beta_1$	$\hat\beta_2$	$\hat\sigma^2_e$	R^2
4	573	−0.0635 (0.026)	0.0338** (0.0034)	0.0042** (0.0005)	0.0051	0.35
5	442	−0.0634 (0.0273)	0.0318** (0.0034)	0.0048** (0.0005)	0.0040	0.39
6	120	−0.1345* (0.0511)	0.0372** (0.0066)	0.0055** (0.0014)	0.0042	0.46
Maturity stage	n	$\hat\beta_0$	Month 3 $\hat\beta_1$	$\hat\beta_2$	$\hat\sigma^2_e$	R^2
5	69	−0.1062 (0.085)	0.033* (0.012)	0.0053* (0.0016)	0.0052	0.3753
6	591	−0.0468 (0.0246)	0.034** (0.003)	0.0041** (0.0005)	0.0039	0.3434

*$p < 0.05$
**$p < 0.01$
X_1 = average length over husk
X_2 = total length of five kernel rows

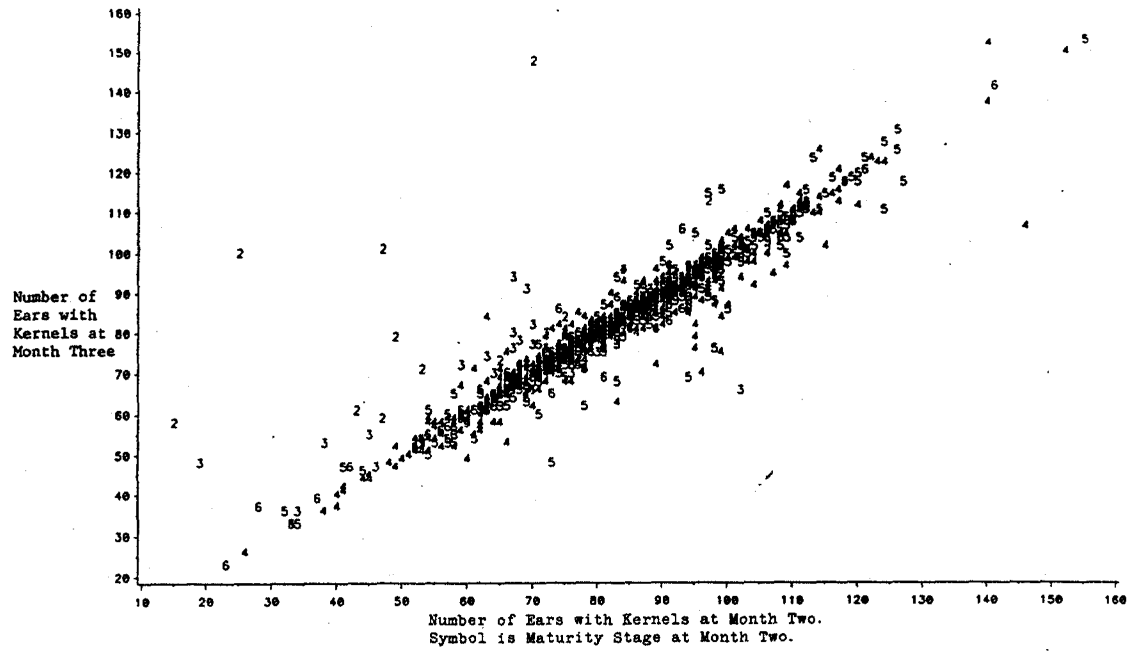

Fig. 1a. Plot of number of ears with kernels at Month Three vs. number of ears with kernels at Month Two

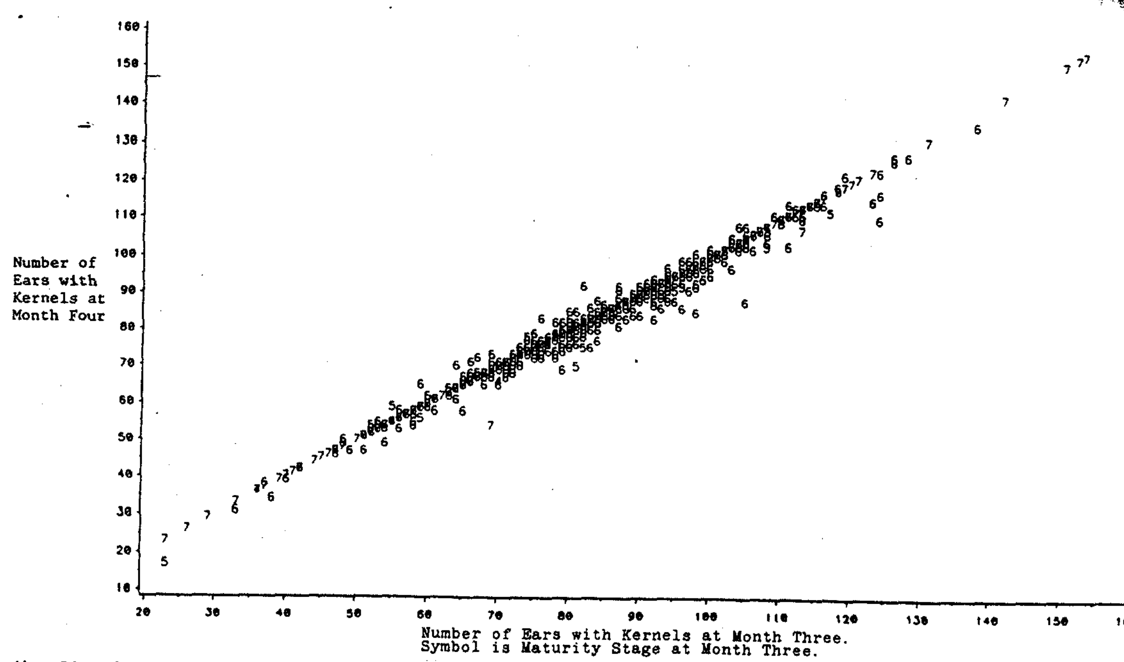

Fig. 1b. Plot of number of ears with kernels at Month Four vs. number of ears with kernels at Month Three

the sample is such that the data are almost self-weighting (Francisco, Fuller, and Fecso 1987), especially in Iowa where the stratification is less distinct than elsewhere. Year-to-year effects do appear to be present in the data, primarily for grain weight, but incorporating them into the estimators also produces only small differences in the values of the estimates. See Reiser, Fecso, and Taylor (1987) for a nested error model of year-to-year effects.

3. Measurement Error

An implicit assumption of the ordinary least squares models presented above is that the regressor variables, the X variables, are fixed in repeated sampling. In reality, the X variables are stochastic, and may be subject to measurement error. Unaccounted measurement error in the regressors generally attenuates the magnitude of the estimated regression slopes, especially in smaller samples. Also, presence of measurement error introduces the possibility of autocorrelated error – i.e., any effect that introduces measurement error of X at one point in time may introduce correlated error in the measurement in X at other times. If X measured at a later point in time becomes the dependent variable in the forecast model, as here, then the equation error term may be correlated with the regressors, a condition under which least squares estimators are not consistent. In the next section we examine measurement error in variables representing number of ears, and in the following section we examine measurement error in variables representing grain weight.

4. Panel Model for Number of Ears

The model presented in this section is a single indicator panel model that will allow us to identify the measurement error in some of the predictors for the count of number of ears per sample unit, under the assumption that error variance is constant over time. A consequence of the rapid growth of corn across a fairly short growing season is that not all variables are available in each month of the Objective Yield Survey, and of those available, some may not be available on all units. The longest span of time within each year over which new observations are available for any of the predictor variables is, as shown in Table 1, only three months. An important variable for which data are available from months two, three and four is the count of ears with evidence of kernel formation.

A bivariate plot is shown for number of ears with kernels in Figure 1. The data in the figures have been edited for lost units, destroyed fields, refusals, etc. An examination of the plot shows that the counts of number of stalks with kernels are quite stable from month to month. Data for other variables show that the count for number of stalks is also very stable across months, but the counts for number of stalks with ears and number of ears or ear shoots are unstable from month one to month two. These differences in stability will also be apparent in the results from the formal model considered below.

A model with measurements at three points in time is shown in Figure 2. We can write the model as follows

$$x = \Lambda_x \xi + \varepsilon \quad (4)$$

and

$$\xi = \beta\xi + \zeta \quad (5)$$

where

x = realized value of random vector X, elements of which correspond to counts for number of ears with kernels at months two, three, and four, (August, September, October)

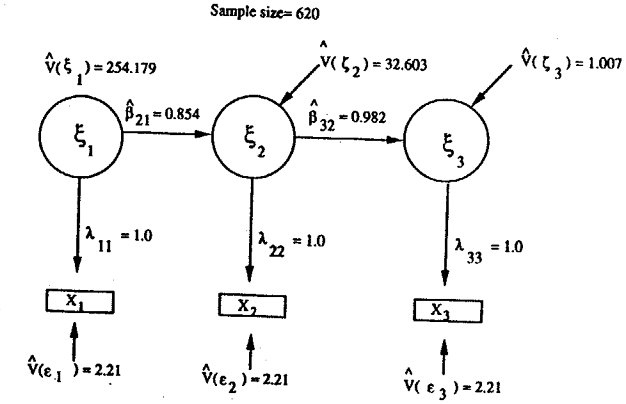

Fig. 2. Single indicator model for number of ears with kernels

ξ = vector of unobservable true values for counts of number of ears with kernels,

Λ_x = matrix of regression parameters that relate observed values to true values,

β = matrix of regression parameters for structural relationships among the true values,

ε = vector of measurement errors,

and

ζ = vector of equation errors.

Expressions (4) and (5) specify relationships among the variables, specifically the ability to predict the next measurement from preceeding measurements, for the purpose of identifying measurement error. That is, X contains a single variable, sometimes referred to as a single indicator, measured at three different points of time. This model has been studied by Hiese (1969), Wiley and Wiley (1970), Wiley (1973), and Jöreskog and Sörbom (1989). See also Munck (1991) for an application of a structural model with measurement errors to survey data.

We do not consider separate models by maturity class in this panel model approach, because it would not be feasible to do so. The maturity stage for a field changes across the growing season, but the changes are irregular. Of the fields that are at stage two in month two, some move to stage three at month three, while others move to stage four. Therefore, it would not be useful to track a particular class of corn based on month two observations. As discussed below, the observations used with this model are mostly from corn that was at maturity stage two, three or four during month two.

For the model shown in Figure 2, the following specifications are also required in order to identify the model

$$\Lambda_x = I$$

$$\beta = \begin{pmatrix} 0 & 0 & 0 \\ \beta_{21} & 0 & 0 \\ 0 & \beta_{32} & 0 \end{pmatrix}$$

$$\xi_1 = \zeta_1,$$

and

$$V(\varepsilon_1) = V(\varepsilon_2) = V(\varepsilon_3) = V(\varepsilon).$$

The last expression states the assumption that the measurement error variances are constant across time.

Using these specifications, the model is *just identified*, and the parameter estimates can be calculated directly from the covariance matrix, following Wiley and Wiley (1970)

$$\begin{aligned}
\hat{\beta}_{32} &= \widehat{\mathrm{Cov}}(X_1, X_3)/\widehat{\mathrm{Cov}}(X_1, X_2) \\
\hat{V}(\varepsilon) &= \hat{V}(X_2) - [\widehat{\mathrm{Cov}}(X_2, X_3)/\hat{\beta}_{32}] \\
\hat{V}(\zeta_1) &= \hat{V}(X_1) - \hat{V}(\varepsilon) \\
\hat{\beta}_{21} &= \widehat{\mathrm{Cov}}(X_1, X_2)/\hat{V}(\zeta_1) \\
\hat{V}(\zeta_2) &= \hat{V}(X_2) \\
&\quad - [\hat{\beta}_{21}\widehat{\mathrm{Cov}}(X_1, X_2) + \hat{V}(\varepsilon)] \\
\hat{V}(\zeta_3) &= \hat{V}(X_3) \\
&\quad - [\hat{\beta}_{32}\widehat{\mathrm{Cov}}(X_2, X_3) + \hat{V}(\varepsilon)]
\end{aligned} \tag{6}$$

Under the assumption of multivariate normality, these expressions are maximum likelihood estimators.

Six hundred and twenty observations are used in the analysis presented below, and they constitute a little less than one-half of the 1 440 fields selected for observation from 1979 to 1985. Approximately 200 fields were lost to refusals, damage to crop, etc. The other 600 fields are not included in the analysis because the model requires month four observations, and the crop was already harvested by the time the USDA enumerator visited the field. Therefore, the observations used for this analysis are from fields that were slow to mature, which implies a low maturity stage at month two. In the presentation of our results, we will comment on results that might have been different with corn that would have been faster to mature.

Because data from seven years were combined to obtain the sample size of 620, year-to-year variation is present in these data. The most likely year-to-year differences would be in the means. Although there is no model on the means in this analysis, different means might imply that covariances among the variables differ over years. In such a case, separate models could be estimated for several years simultaneously, as will be done in a later section of the paper. Here there is no evidence that covariances differ across years, and a single model will be retained.

Estimates for model parameters are given in Figure 2, where it is apparent that the magnitude of the measurement error variance, at 2.21, is very small. The reliability ratio, ρ^2, is defined as the ratio of variance of the true measure to the total variance

$$\rho^2 = V(\xi)/[V(\xi) + V(\varepsilon)]. \tag{7}$$

Under this definition, the reliability ratio of the measurements at times one, two and three are as follows

$$\rho_1^2 = 0.991$$

$$\rho_2^2 = 0.990$$

$$\rho_3^2 = 0.990$$

Although it is not necessarily the case that the reliabilities be equal across time points, it is clear that number of ears with kernels is measured virtually without error at months two, three and four. This result is not surprising, given the relationships shown in Figures 1a and 1b. Since these measurement error variances are so small, it is natural to test the null hypothesis that they are equal to zero. If we adopt the assumption that

$$\begin{pmatrix} x \\ \xi \\ \zeta \\ \varepsilon \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_x \\ \mu_\xi \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} \Sigma_{xx} & \Sigma_{x\xi} & 0 & 0 \\ \Sigma_{\xi x} & \Phi & 0 & 0 \\ 0 & 0 & \Psi & 0 \\ 0 & 0 & 0 & \Theta_\varepsilon \end{pmatrix}\right),$$

where Ψ and Θ_ε are diagonal (i.e., all errors are uncorrelated), then we will be able to perform a likelihood ratio test of this null hypothesis by using a general purpose computer package such as LISREL (Jöreskog and Sörbom 1989), or LISCOMP (Muthén 1988). Under the constraint that these measurement error variances are equal to zero, the model fits poorly ($p < 0.05$), indicating that the variances are significantly different from zero. This likelihood ratio test requires an assumption that sampled fields were selected by simple random sampling, not complex sampling which might involve stratification or clustering. As discussed earlier, the "design effect" of the complex sample used to collect this data is essentially equal to 1.0, so the assumption of simple random sampling is not unreasonable. Methods for covariance structure models that do not assume simple random sampling are now becoming available. See Satorra (1991).

If there were no measurement error, the emergence and persistence of number of ears with kernels across three measurements made during the growing season would conform exactly to a Markov simplex process. Instead, because of the presence of measurement error, the growth of the ears follows a quasi-Markov process. In the quasi-Markov process, the true value for number of ears with kernels at month i, given the true number at month i-1, is independent of the true number at any other month. (Anderson 1959; Jöreskog 1970a.) The variance of the disturbance term in the structural equations changes considerably from month two ($\psi_{2,2} = 32.60$) to month three ($\psi_{3,3} = 1.01$), so the process of growth of ears is non-stationary.

As mentioned earlier, the model shown in Figure 2 is *just identified*, a term that refers to the relationship between the parameters and the information in the variance-covariance matrix implied by the model. *Just identified* implies that a unique value can be obtained for each parameter, but there are no degrees of freedom left for a " test of fit." It is not, however, a liability for a model to be just identified; we can interpret the magnitude of the parameter estimates, and we often test that parameters are equal to zero. We have done interpretations of and tests on parameters here and we found that the measurement error variance, although small, is significantly different from zero.

We fit this simple model to variables that are measured in months one, two and three, instead of months two, three and four, which are the months for number of ears with kernels. For the count of number of stalks, the reliability ratio had the value 0.998 in each of the three months. However, the model was not successful for the measurement of stalks with ears or for the measurement of ears/ear shoots. For each of these variables, the estimated error variance was negative by a substantial amount. We interpret this result as an indication that the assumption of constant error variance, and hence the model, is not appropriate. Bivariate plots show that number of ears with kernels and number of stalks are quite stable from month to month. Number of ears and number of stalks with ears, on the other hand, are very unstable early in the season, as seen by comparing month one to month two.

It is clear that the larger measurement errors at month one in the counts of stalks with ears and ears or ear shoots are not errors of counting. Obviously, many ears that are counted at months two and three have simply not emerged yet at month one. Also, ear shoots that begin to form may ultimately die off. The model used in this section allows for change in the number of ears at each sample location, but the change must be reliable, i.e., predictable. So, there appears to be a period of inherent unreliability in the timing of emergence of ears.

5. A Two-Variable, Two-Wave Model for Size of Ears

Two variables are used as predictors of grain weight: average length over husk (X_1), and total length of five kernel rows (X_2). Bivariate plots of these variables across months are shown in Figure 3 and 4, where it appears that length of kernel row contains more error. The size of an ear of corn should, of course, be represented by at least length and diameter (or circumference), as if its shape were a cylinder. Using simple geometric shapes for modeling has worked well for fruit crops (Fecso 1975). Recently Bigsby (1989) found that including diameter measures for corn reduced the mean squared error by 30% to 50% from models with only length measures. However, there are no measures of diameter or circumference available in this data, thus we will examine only measures of length.[4]

Neither of the X variables is an ideal measurement: we would like to have a measurement of grain weight at each month of the growing season, but obtaining it would destroy the ear being measured. So, the length of the cob over the husk is measured instead. To get a measurement that is closer to the actual grain weight, the length of the kernel row is measured for five ears. But again, since this measurement destroys the ear in terms of future growth, it is done on five ears that are *outside* the unit, and on plants that are different from those for which grain weight will be determined at the end of the season. Thus, there may be substantial error in both manifest variables as measures of the same underlying variable.

Neither of the variables for grain weight are measured for three consecutive months, so the model given in the previous section cannot be used. However, since both variables are available in months two and three, the two-variable two-wave model shown in Figure 5 can be used instead. The general form of this model is the same as the form of the model in equations (4) and (5), namely

$$x = \Lambda_x \xi + \varepsilon,$$

and

$$\xi = \beta\xi + \zeta,$$

but now

X_1 = average length over husk at month two

X_2 = average length of five kernel rows at month two

X_3 = average length over husk at month three

X_4 = average length of five kernel rows at month three

ξ_1 = true (average) length of corn cobs at month two

ξ_2 = true (average) length of corn cobs at month three.

Identification conditions for this model are discussed by Wiley (1973). If we impose additional restrictions,

$$\Lambda_x = \begin{pmatrix} 1 & 0 \\ \lambda_{21} & 0 \\ 0 & 1 \\ 0 & \lambda_{42} \end{pmatrix},$$

$$\xi_1 = \zeta_1$$

then all parameters are identifiable in terms

[4] For the analysis in this section, we use USDA variable P19 divided by 5.0, to give average length over the five kernel rows. This transformation was done so that the metrics of the two X variables would be approximately in the same units.

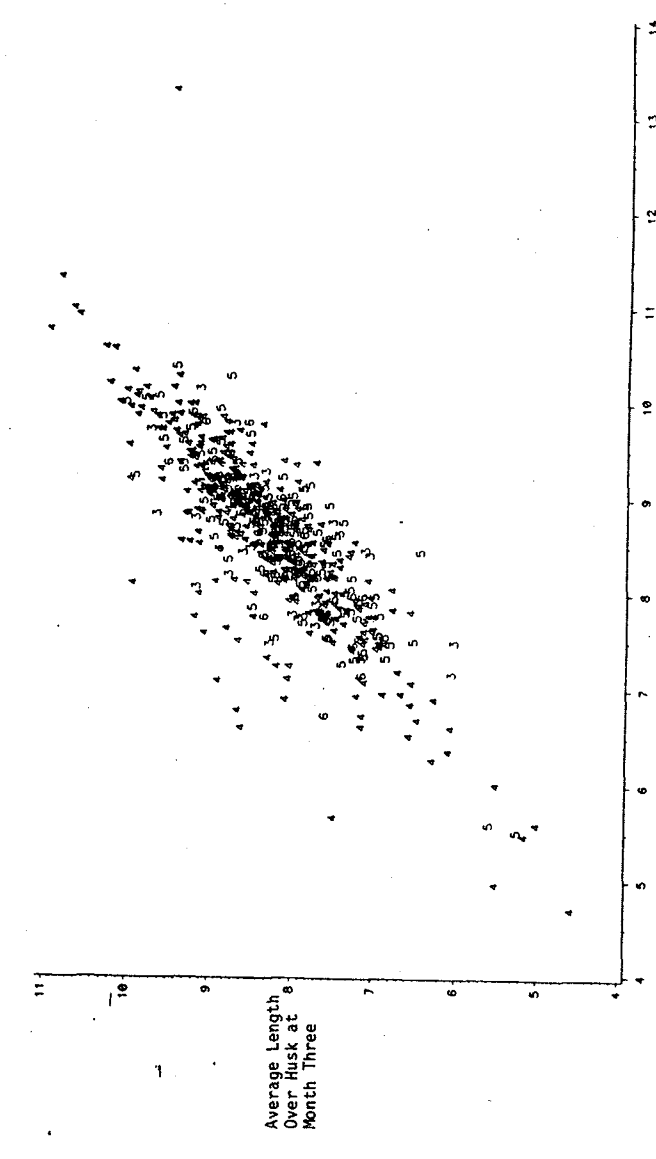

Fig. 3. *Plot of average length over husk at Month Three vs. average length over husk at Month Two*

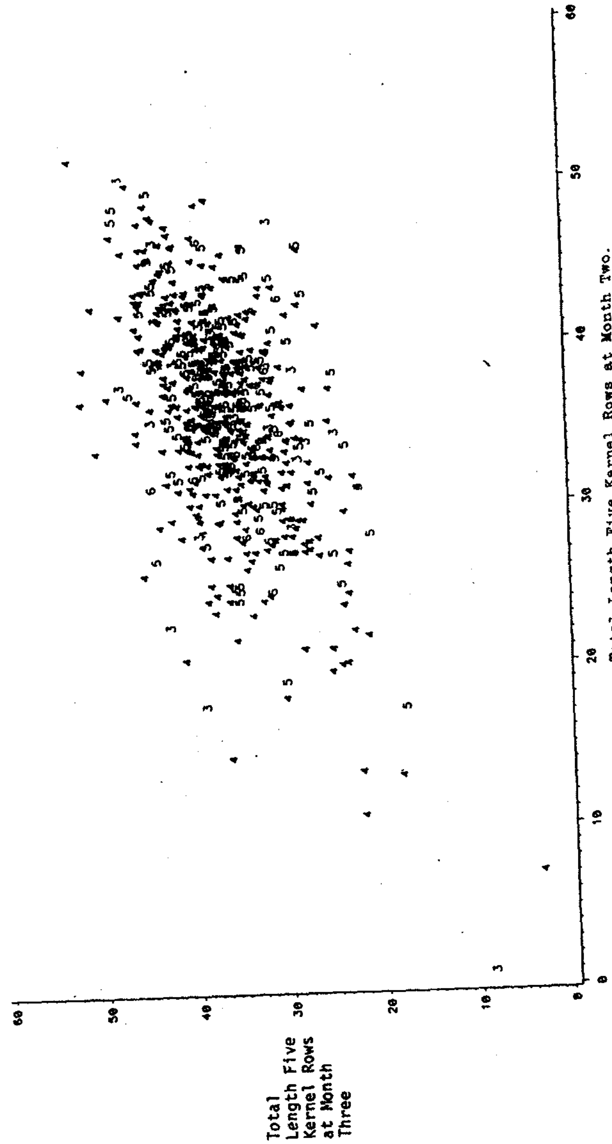

Fig. 4. Plot of total length for five kernel rows at Month Three vs. total length for five kernel rows at Month Two

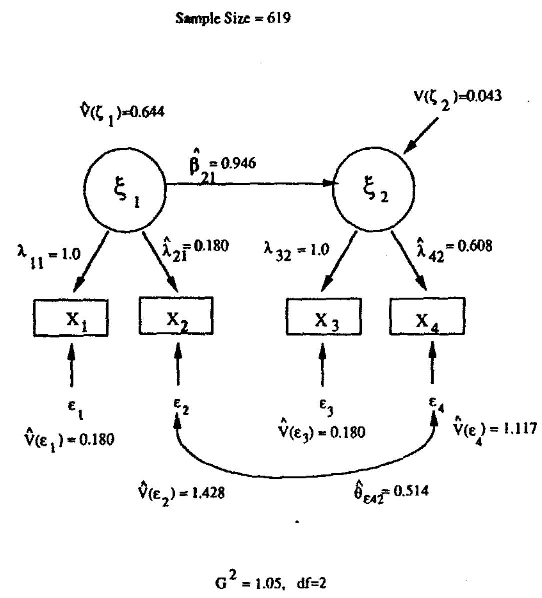

Fig. 5. Two-variable, two-wave model for size of ears

of the covariance matrix among the X variables.

We let ω represent a vector containing all parameters of the model, i.e., ω contains the elements of β, Λ, and the non-redundant elements of Φ, Ψ, and Θ_ε. Then we can refer to $\Sigma(\omega)$ as the covariance matrix implied by the specified model. In this case

$$\Sigma(\omega) = \Lambda(I - B)^{-1}\Psi(I - B)^{-1}\Lambda' + \Theta_\varepsilon$$

$$= \begin{pmatrix} \lambda_{11}^2\Psi_{11} & & & \text{sym} \\ \lambda_{11}\lambda_{21}\Psi_{11} & \lambda_{21}^2\Psi_{11} & & \\ \lambda_{11}\lambda_{31}\beta_{21}\Psi_{11} & \lambda_{21}\lambda_{31}\beta_{21}\Psi_{11} & \beta_{21}^2\lambda_{31}^2\Psi_{11} + \lambda_{31}^2\Psi_{22} & \\ \lambda_{11}\lambda_{41}\beta_{21}\Psi_{11} & \lambda_{21}\lambda_{41}\beta_{21}\Psi_{11} & \beta_{21}^2\lambda_{41}\lambda_{31} + \lambda_{41}\lambda_{31}\Psi_{22} & \beta_{21}^2\lambda_{41}^2\Psi_{11} + \lambda_{41}^2\Psi_{22} \end{pmatrix} + \Theta_\varepsilon$$

Setting $\Sigma(\omega) = \Sigma_{yy}$ (i.e., setting the covariance matrix implied by the model equal to the actual covariance matrix), the expression given above produces ten simultaneous equations in nine unknown parameters. From these simultaneous equations, it follows that $\lambda_{21} = \sigma_{23}/\sigma_{13}$ and also that $\lambda_{21} = \sigma_{24}/\sigma_{14}$. Hence the model implies that

$$\sigma_{14}^2\sigma_{23}^2 = \sigma_{13}^2\sigma_{24}^2. \quad (8)$$

Given ten equations and nine unknowns, there is one degree of freedom to test this constraint.

Moment estimates of the parameters may be calculated directly from the sample covariance matrix, but if we adopt the assumptions discussed in Section 4, then maximum likelihood estimates, as well as estimated asymptotic standard errors, may be obtained from LISREL (Jöreskog and Sörbom 1989) or LISCOMP (Muthén 1988). Under these assumptions, we will also be able to perform a likelihood ratio test of the constraint given in expression (8). Statistical properties of this model are well known (Jöreskog 1970b).

The model as given above shows a poor fit to the data, since the likelihood ratio statistic is 98.78 on one degree of freedom. Such a poor fit is usually taken as an indication that correlation of errors across occasions is present (Kessler and Greenberg 1981). In addition, residuals given by the LISREL program suggest that the covariance between length of kernel row at month two and length of kernel row at month three ($\hat{\sigma}_{2,4}$) is not very well replicated under the model. Lagrange multiplier statistics produced by LISREL suggest that setting element 2,4 in Θ_ε as a free parameter to be estimated would dramatically improve the fit of the model. (In this first model, which we refer to as model A, it is constrained to zero by default.)

In Table 4, results for the model with this covariance as a free parameter are shown under model B. These results are also shown in Figure 5. Model B also contains equality constraints on other parameters, namely, $\lambda_{21} = \lambda_{42}$ and $\Theta_{\varepsilon 1,1} = \Theta_{\varepsilon 3,3}$. These are reasonable constraints to include in the model, and in this case they appear to reflect the data accurately, since the model fits well with a likelihood ratio statistic of $G^2 = 1.05$ on two degrees of freedom. The two constraints given above also serve another purpose: without them Ψ_{22} is slightly negative (i.e., outside the parameter space). Since such a parameter value is unacceptable, the model is in need of modification, and the equality constraints provide appropriate modifications.

As is evident from Table 4, the amount of error in the measurement of lengths is large. These error variances correspond to the following reliabilities

$$\rho^2_{x1} = \rho^2(X_1) = 0.73$$

$$\rho^2_{x2} = \rho^2(X_2) = 0.16$$

$$\rho^2_{x3} = \rho^2(X_3) = 0.78$$

$$\rho^2_{x4} = \rho^2(X_4) = 0.16.$$

Clearly, average length over husk (X_1, X_3) appears to be the preferable measure. In assessing these results, it is important to keep in mind that measurement error encompasses not only errors that literally occur with a tape measure in the field, but also components of the measured variable that are unrelated to the true value. That is, there appears to be some systematic variance in length of kernel rows (X_2 and X_4) that is unrelated to average length over husk (X_1 and X_3), and may be unrelated to end-of-season grain weight. Psychometricians use the term parallel measurements for two variables if their latent variables are linearly dependent and their measurement errors are independent, with equal variances. Clearly, length of kernel rows and average length over husk are not parallel measurements.

An important difference between these two variables is that length of kernel rows, since it is a destructive measure, is taken on the first five ears *outside* the unit. Also, since the ears used for the measurement in month two are destroyed, length of kernel row has to be measured on five different ears in month three. The ear-to-ear variability affects its reliability significantly, vis-a-vis length over husk which is measured inside the unit.

Table 4a. Parameter estimates for two-variable, two-wave model B (Estimates for model A not shown)

Parameter	Model B Estimate (s.e.)	
β_{21}	0.946	(0.050)
λ_{21}	0.608	(0.055)
λ_{42}	0.608	(0.055)
$\Psi_{11}(=\Phi_{11})$	0.644	(0.054)
Ψ_{22}	0.043	(0.050)
$\theta_{\varepsilon 11}$	0.180	(0.028)
$\theta_{\varepsilon 22}$	1.428	(0.083)
$\theta_{\varepsilon 33}$	0.180	(0.028)
$\theta_{\varepsilon 44}$	1.117	(0.066)
$\theta_{\varepsilon 42}$	0.514	(0.057)
	$G^2 = 1.05$	
	$df = 2$	

Table 4b. Parameter estimates for two-variable, two-wave model C

	Model C 1979, 1981 1982, 1985	
Parameter	Estimate (s.e.)	
β_{21}	0.909	(0.081)
β_{32}	0.043	(0.004)
λ_{21}	0.505	(0.060)
λ_{42}	0.505	(0.060)
$\Psi_{11}(=\Phi_{11})$	0.529	(0.060)
Ψ_{22}	0.095	(0.050)

	1979		1981		1982		1985	
Parameter	Estimate (s.e.)		Estimate (s.e.)		Estimate (s.e)		Estimate (s.e.)	
$\theta_{\varepsilon 11}$	0.209	(0.040)	0.160	(0.065)	0.183	(0.060)	0.043	(0.057)
$\theta_{\varepsilon 22}$	1.258	(0.152)	1.889	(0.251)	0.867	(0.103)	1.416	(0.187)
$\theta_{\varepsilon 33}$	0.209	(0.040)	0.099	(0.049)	0.136	(0.047)	0.163	(0.049)
$\theta_{\varepsilon 44}$	1.105	(0.134)	1.103	(0.147)	0.990	(0.117)	1.002	(0.133)
$\theta_{\varepsilon 42}$	0.434	(0.108)	0.729	(0.152)	0.109	(0.078)	0.568	(0.007)
$\theta_{\varepsilon 52}$	0.022	(0.006)	0.013	(0.008)	0.0004	(0.004)	0.023	(0.007)
$\theta_{\varepsilon 54}$	0.018	(0.006)	0.022	(0.006)	0.013	(0.005)	0.013	(0.005)
$\Psi_{33}(=\theta_{\varepsilon 55})$	0.004	(0.0005)	0.004	(0.0005)	0.003	(0.0005)	0.003	(0.0005)
			$G^2 = 27.08$					
			$df = 24$					

Table 4c. Reliability ratios by year

	X_1	X_2	X_3	X_4
1979	0.703	0.078	0.746	0.056
1981	0.776	0.035	0.852	0.103
1982	0.711	0.123	0.799	0.095
1985	0.936	0.169	0.793	0.211

Results for this model were obtained using data from the second and third months of the Objective Yield Survey. One aspect of the data that should be kept in mind is that month three data regarding length of the cob are available only for the plants that were the slowest to mature. If the crop was already mature at month three, which was the case for just over one half of the sample locations, the corn would have been harvested and no measurements would have been taken regarding length of the cobs.

In model B, the covariance between the measurement errors in length over husk from month one to month two ($\Theta_{\epsilon 13}$) is fixed at 0.0 by default. Normally, this parameter is not identified in the two-wave, two-variable model. When the equality constraints $\lambda_{21} = \lambda_{42}$ and $\Theta_{\epsilon 11} = \Theta_{\epsilon 33}$ are included in the model, then $\Theta_{\epsilon 13}$ is identifiable and may be specified in LISREL as a free parameter to be estimated. Our attempts to estimate $\Theta_{\epsilon 13}$ gave results that indicate severe multicolinearity in the sampling distribution of the parameter estimators. These results included difficulties in convergence of the numerical solution and huge standard errors for estimates. In such a result, the model is sometimes said to be overparameterized – i.e., there is not enough information in the data to effectively estimate all of the parameters included in the model. So although model B fits very well when $\Theta_{\epsilon 13}$ is fixed at 0.0, we do not interpret that result as very compelling evidence that $\Theta_{\epsilon 13}$ is actually equal to 0.0. An independent replicated measurement of the observation on each ear would provide the added information to effectively estimate $\Theta_{\epsilon 13}$, but such measurements are simply not available in this data, and would in general be expensive to obtain. We discuss another approach for estimating this covariance in Section 6.

One reason for examining measurement error is to assess the effect on parameter estimates used in forecasting models discussed in Section 2. One way to approach this question is to introduce the end-of-season grain weight as a variable into the panel model, as shown in Figure 6. We refer to the model including end-of-season grain weight as model C. Since end-of-season grain weight has only one measurement, we must assume that it is measured without error, or equivalently, that the measurement error is absorbed into the disturbance term.

In estimating model C, we also calculated covariance matrices for each year separately, and found evidence that these within-year covariance matrices are not equal. For the years 1979, 1981, 1982, and 1985, we were able to estimate a two-wave two variable panel model using the separate within-year covariance matrices simultaneously. The method consists of simultaneously fitting separate models for each year, with the possibility of constraints on parameters across years. A single test of fit can be performed for all years. When all parameters were constrained to equality across years, the model showed a poor fit ($G^2 = 84.34$, $df = 47$). If we allowed error variances and covariances to be free across years, then the fit was satisfactory ($G^2 = 26.07$, $df = 23$). For three years, 1980, 1983 and 1984, there were too few observations to include in the simultaneous estimation of separate models.

Parameter estimates are shown by year in Table 4b under model C, and in Figure 6. The largest difference across years occurs in

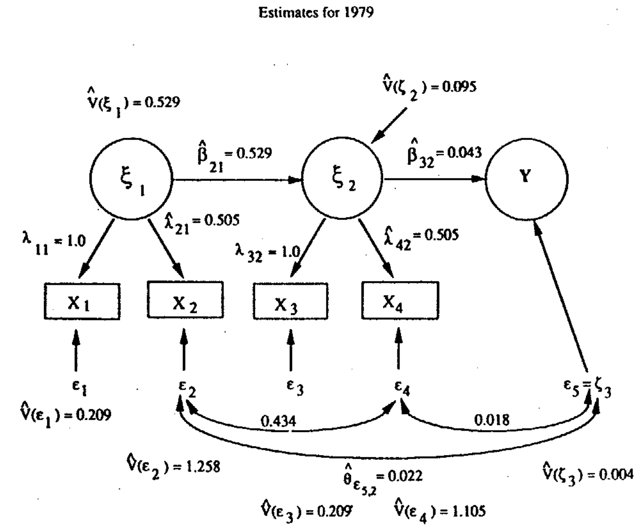

Fig. 6. Model including grain weight (Y)

1982, where $\theta_{\epsilon 42}$ is much smaller than in the other years. Changes in error variances across years imply changes in reliability ratios, which are shown by year in Table 4c. Average length over husk in month three appears to be the most reliable measurement.

Model C can be used to assess the effect of measurement error on estimates that might be used in forecasting end-of-season grain weight. In model C, not only is measurement error present, but we were also able to estimate a covariance between the disturbance term for the structural equation and the measurement error term for length of kernel row. The presence of measurement error and correlated errors may cause ordinary least squares estimators, which are based on the assumption of no measurement error, to be biased or inconsistent. We can investigate the effect of measurement error and correlated errors by comparing estimates for the β parameters from structural equations in model C to estimates under two other conditions: (1) measurement error fixed at zero, and, (2) measurement error present, but error covariances fixed at zero. Estimates for β are inflated when the measurement errors are fixed at zero: $\beta_{21} = 0.930$ and $\beta_{32} = 0.056$, as compared with the values of 0.909 and 0.043 under model C. When measurement errors are present, but error covariances are fixed at zero, β_{21} and β_{32} had estimates of 1.111 and 0.050, respectively. As can be seen from these numbers, ignoring measurement error may cause the estimated regression slope that would be used in a forecast to be inflated by as much as 30%. Even though measurement error may have a substantial effect on the parameter estimate that would be used in forecasting, there are some conditions under which the OLS estimate would still be optimal in forecasting.

This topic will be discussed more fully in Section 7.

In the next section, we consider an alternative approach that will allow us to estimate the entire covariance matrix among the measurement error terms. A model which contains the entire error covariance matrix will be more realistic, and may give additional insight into the effect of measurement error on the estimation of parameters that would be used in forecasting. With the two-wave panel model used in this section, only one error covariance across time can be estimated, unless additional indicators are available at each time point. Since the additional measurements are not available, we consider an approach that equates measurement error variance to within year sampling variance.

As mentioned above, for 1980, 1983 and 1984, there were too few observations to estimate separate models. These were also the years of low end-of-season yield. In order for an observation to be included in this analysis, data had to be available for both months two (August) and three (September). For the three years with too few observations, harvests in most fields must have been completed unusually early, so that by September 1, there were only a few sample fields left in which forecast measurements could be taken. This serves as a reminder that even in the other years when more observations were available, the measurements used to estimate these panel models were obtained from the slower growing plants, because only measurements of at-harvest yield were collected from the faster growing plants during the September survey. Of course, forecasts are not required for plants on which the actual values can be obtained.

6. Measurement Error in Yearly Means

We may specify equations that correspond to equation (2) or (3) and that also include components for measurement error as follows

$$y = \beta_0 + \beta x + e \qquad (9)$$

and

$$x = \xi + \varepsilon \qquad (10)$$

where

x = vector of observed values for predictor variables, measured with error, from a single month

y = realized value for end-of-season yield

ξ = vector of true values for predictor variables

e = equation error

ε = vector of measurement errors.

The covariance matrix of the error terms has the following structure

$$E\{(e, \varepsilon)'(e, \varepsilon)\} = \begin{pmatrix} \sigma_e^2 & \Sigma_{e\varepsilon} \\ \Sigma_{\varepsilon e} & \Sigma_{\varepsilon\varepsilon} \end{pmatrix}. \qquad (11)$$

We envision that this model would be used with predictors from only one month at a time. Expression (10) specifies that each predictor variable is associated with a unique latent variable which represents the true value. In Section 5, expression (5) provided two predictor variables associated with each latent variable. When there are two predictor variables at just one time point, the two approaches are equivalent under the condition that $\beta_1 \equiv \beta_2$ from β in expression (10). Equivalences between the models will be discussed in more detail below. As for the models considered in Section 2, separate values for β_0 and β could be used for each maturity class within month, and the model could be so subscripted. To simplify notation, the additional subscripts are not used here.

When variables are included in the model from only one month, there are only two predictors, and so the measurement error variances or reliability ratios must be assumed known. Estimation of the parameters for this forecasting model could be attempted by using the generalized least squares estimator given by Fuller (1987) along with the reliabilities that are implied by model C. Such estimates may be calculated with either the Super Carp (Hidiroglou, Fuller, and Hickman 1980) or EV Carp (Schnell and Fuller 1987) programs. In order to use this estimator, we must assume that the matrix of sums of squares and cross products for the true values, ξ, is positive definite. The test of singularity is performed on the smallest root of

$$|X'X - \lambda(n-1)D\Lambda_{\varepsilon\varepsilon}D| = 0 \qquad (12)$$

where $D = \text{diag}\,(S_{x1}, S_{x2}, \ldots S_{xp})$ and $\Lambda_{\varepsilon\varepsilon}$ contains ratios of error variance to total variance for the independent variables on the diagonal and the corresponding covariance ratios as off diagonal elements.

If the two predictor variables are congeneric measures, which means that they are representations of the same underlying variable, then the true values have unit correlation. In that case, the matrix of sums of squares and cross products for the true value will be singular. With two predictor variables at a single point in time, the predictors are congeneric measures if and only if $\beta_1 \equiv \beta_2$ and the measurement error variances are uncorrelated. Given the successful model for congeneric measures in the preceding section, it comes as no surprise that if we use the same data as in Section 5 as well as error ratios calculated from model C, then the test of singularity is failed for all months and maturity codes. This result implies that there is only a single parameter to be estimated (either β_1 or β_2, but not both), and the value of the estimate is identical to $\hat{\beta}_{12}$ of model C in Section 5.

From these results, it is evident that the covariances among the measurement error terms within a time point play an important role in determining a forecasting model. Unfortunately, the estimation of these covariances within model C required additional indicators at each time point that were not available. In order to obtain values for all of the error covariances, we used an entirely different method for identifying the error covariance matrix, following an example in Fuller (1987, p. 131). In this method, using all seven years of data, the error covariance matrix was estimated by pooling the seven sample covariance matrices for the variation among units within years, and then dividing by n_0, where

$$n_0 = \frac{1}{7-1}\left(N - \frac{\Sigma n_i^2}{N}\right) = 82.44$$

represents the number of observations per year (See Snedecor and Cochran 1967, p. 290). That is, the measurement error covariance matrix was equated with the covariance matrix of the sampling distribution of the *yearly means* of the five variables Y, X_1, X_2, X_3, and X_4, where X_1, X_2, X_3, X_4 were defined in Section 4, and Y is end-of-season grain weight.

Table 5a gives the sample covariance matrix for the five variables calculated from yearly means on seven observations (i.e., seven years), and Table 5b gives the error covariance matrix based on the pooled within year covariance matrix as described above. The variances from the two matrices were used to calculate the following reliability ratios

$$\rho^2(X_1) = 0.914$$

Table 5a. *Covariance matrix among yearly means, df = 6*

	X_1	X_2	X_3	X_4	Y
X_1	0.103				
X_2	0.009	0.089			
X_3	0.117	0.097	0.149		
X_4	0.098	0.088	0.114	0.099	
Y	0.013	0.009	0.015	0.012	0.002

Table 5b. *Error covariance matrix based on the pooled within year variation*

	X_1	X_2	X_3	X_4	Y
X_1	0.0089				
X_2	0.0038	0.0190			
X_3	0.0063	0.0038	0.0086		
X_4	0.0032	0.0090	0.0034	0.0152	
Y	0.0003	0.0004	0.0003	0.0004	0.00006

$df = 595$ $$n_0 = \frac{1}{a-1}\left(N - \frac{\Sigma n_j^2}{N}\right) = 82.44$$

(See Snedecor and Cochran, 1967, p. 290)

$$\rho^2(X_2) = 0.786$$
$$\rho^2(X_3) = 0.943$$
$$\rho^2(X_4) = 0.846$$
$$\rho^2(Y) = 0.971.$$

These values are considerably higher than the estimates obtained from the panel model in Section 5, especially for length of kernel row (X_2, X_4). The values follow from the definition of reliability given earlier; the difference between these values and the estimates from Section 5 is due to different definitions of error variance and true variance. In the panel model, the estimate of true variance is based essentially on correlation across months of the growing season, whereas for the yearly means true variance is a function of sample size. Given the large samples, the yearly means have higher reliabilities than the monthly measurements.

The matrix in Table 5b contains values for the error covariances that were not identified in the panel model of Section 5. From these values, it appears that the error correlation between X_1 and X_3, at 0.72, is even larger than the error correlation between X_2 and X_4, which is 0.53. The other error correlations are also fairly large. These results for error correlations represent an addition to the panel model results in Section 5. As stated before, there is not enough information in the two-wave two-variable panel model to estimate all error covariances, and so some of them were fixed at zero by default. The method used in this section shows that all error covariances are considerably different from zero.

Using the seven observations for yearly means, and the error covariances in Table 5, we can again attempt to use Fuller's estimator to obtain sample values for the parameters of the errors-in-variables model in expressions (9) and (10). Unfortunately, length over husk and length of kernel row are highly colinear, and with only seven observations, we cannot reject the null hypothesis that the matrix of sum of squares and cross products for the true values is singular. Therefore, it would not be appropriate to rely on the estimated values obtained for the model including both these variables. If we use only one predictor variable, and choosing length over husk at month three (X_3) to be that variable, the results appear reasonable: $\hat{\beta}_1 = 0.1038$ with measurement errors equal to zero and $\hat{\beta}_1 = 0.1065$ using the appropriate error variances from Table 5. Both values are significantly different from zero, with $p < .01$. So, the effect of measurement error is very modest here, but the magnitude of the estimated regression slope is approximately double the value obtained with the panel model approach. The difference is due to the focus on yearly means as units of analysis rather than measurements from individual fields.

7. Discussion and Conclusions

Measurement error in counts for number of ears and number of stalks appears to be minimal during the Objective Yield Survey. In month one, the number of ears can be predicted well from number of stalks, and since number of stalks is measured with a high reliability, number of ears can be predicted with a low error even at month one. In the second and third months, counts for number of ears are very reliable, and the forecast for number of ears can be made with even lower error. Measurement error in the counts for number of ears is not an important aspect of the Objective Yield Survey for corn. Counting may be more subject to error with other crops such as soybeans.

Grain weight is inherently more difficult to measure than number of ears. Because the ear of corn must be destroyed to make the grain weight measurement, proxy variables must be used during the Objective Yield Surveys, which introduces measurement error that makes a large contribution to the error in forecasting end-of-season yield. Indicators for size of ear were studied over months two and three, when month-to-month magnitudes are fairly stable. Length over cob, measured on the same plants over time, appears to have a reasonably high reliability ratio of 0.76. On the other hand, length of kernel row, which is measured outside the unit, has a much lower reliability. The reliability of this variable could be substantially improved by increasing the number of ears measured for length outside the sampling unit in the field. This increase would entail little cost, since the size of the sample unit would not have to be increased, and no additional laboratory work would be required. The larger sample of ears would lead to a higher reliability, a reduction in the standard error for the estimated regression coefficient, and a reduction in the standard error of the forecast.

In this paper we have used statistical models to assess measurement error in the Objective Yield Survey. Error in the measurement of predictor variables reduces precision of the prediction, and may result in an estimator that is not consistent. When the purpose of a model is prediction, as in the Objective Yield Survey, the OLS estimator may still be optimal. If the prediction is for a random element *from the same distribution* as the other X and Y values, and the variables are multivariate normal, then the optimal prediction can be obtained by the ordinary least squares estimator even in the presence of non-zero covariance between the equation error and the measurement error. If these conditions are not met, then it would be desirable to give a prediction which would be conditional on the true value of the predictor. (See Fuller 1987, pp. 74–79.)

The most important condition to consider for the USDA forecasts is whether or not measurements taken from fields for prediction in the current year can be considered to be random elements from the same distribution as the measurements used to establish forecasting parameter estimates. Fields in a current year are clearly sampled from the same (or essentially the same) population as in past years, but sampling with respect to time must also be considered. Years chosen for establishing the model as well as the current forecast year are not selected randomly, and might even be represented by a fixed effect. Moreover, growing conditions in the current forecast year may differ substantially from the conditions in the years used to establish the model. Extended cold weather or severe lack of rain does not usually occur during a growing season, but when it does, the measurements from that year cannot be considered to have been taken from the same distribution as measurements from the previous years, unless they too contained unusual growing conditions. Thus, using the structural model given in expressions (9) and (10) as the prediction equation should be considered by the USDA.

Ultimately the USDA is interested in predicting the yearly total harvest, which can be calculated directly from the yearly mean. Results in Section 6 showed that measurement error would have a very small effect on a direct prediction of the yearly mean. However, the USDA does not use a direct prediction. Instead, a forecast is made for each sample field, and then an average is taken. At the field level, measurement error has a substantial effect on estimated parameters in the forecasting model.

Unfortunately, a complete set of parameter estimates for the true measurement error model at the field level is not available from results in this paper, because there is not enough information in a two-variable two-wave model to estimate the entire covariance matrix for the measurement error terms. While a forecasting model based on results in Section 5 could be used on an interim basis, a more thorough assessment of measurement error in grain weight is needed, and it would require additional indicators of length of cob, so that the appropriate error covariance matrix could be estimated. Measurements of diameter or circumference of the ear of corn should also be included in such a study.

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